# A Fast Lightweight Approach to Origin-Destination Traffic Matrix Estimation through Partial Measurements

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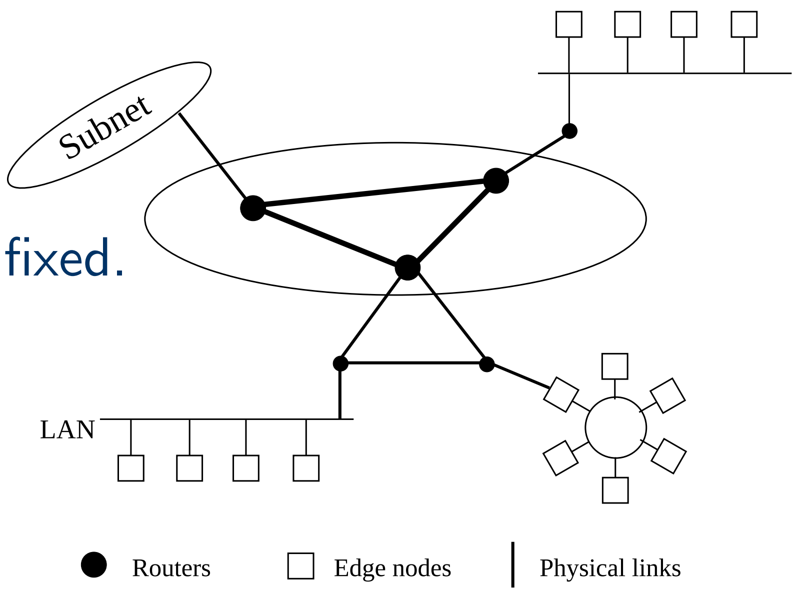
# Outline

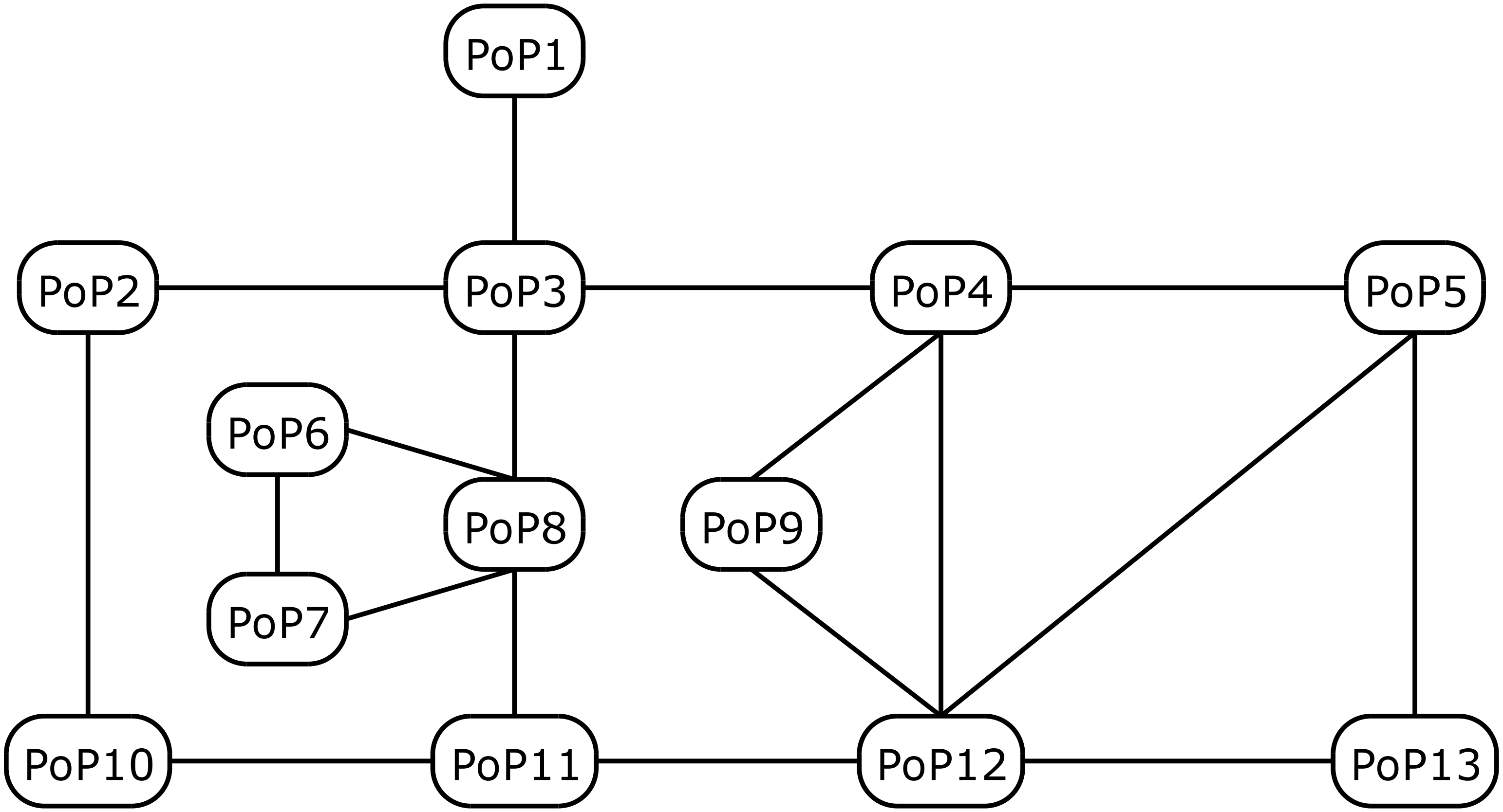
- Introduction
  - Network Terminologies
  - Origin-Destination Traffic Matrix Estimation
- A Partial Measurement Approach (PamTram)
  - A Dynamic Network Traffic Model
  - Traffic Estimation: Iterative Proportional Fitting
  - Pair Selection: Randomization Rules
- Experiment Results (Sprint European PoP Network)
- Conclusions

# Network Terminologies

- Network components:
  edge nodes (origins and destinations), routers, links.
- Network is like a postal system.
  - packets are like letters;
  - edge nodes are like senders and receivers;
  - routers are like mailmen.
- Routing: by destination (IP) address, mostly fixed.



# A Sprint European Pop Network




PoP: Point of presence. 13 PoPs and 18 links.

# Network Management Tasks

Network engineering intends to

- Evaluate and monitor network performance;
- Improve performance based on current network infrastructure.

In order to fulfill such tasks, we need to observe

- Link packet loss probability
- Link delay
- Origin-Destination (OD) traffic matrix
- Topology/connectivity discovery
- . . .

*Usually, there are difficulties in measuring these quantities directly.*

# Origin-Destination Traffic Matrices

- What are Origin-Destination matrices?

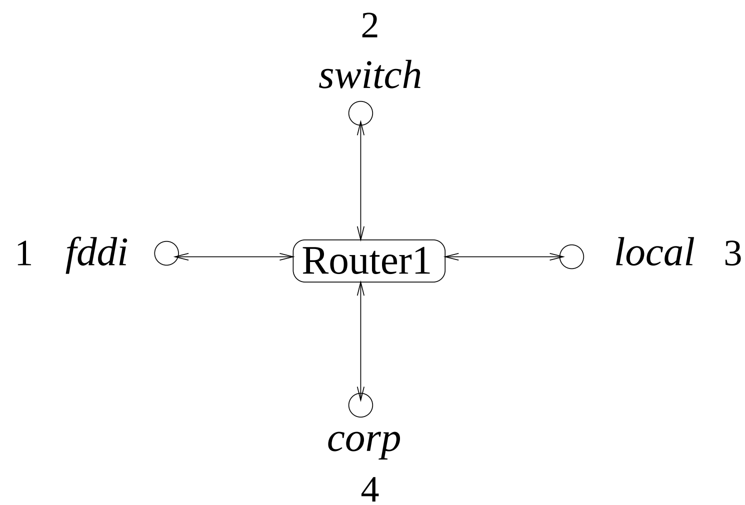
  For a network, Origin-Destination (OD) matrices are the volume of the traffic between all possible pairs of edge nodes in a given time period.

- Why OD traffic matrices?

  OD traffic matrices are important inputs to the network routing algorithms.

OD traffic matrices are in practice hard to observe directly, but link traffic can be easily obtained.

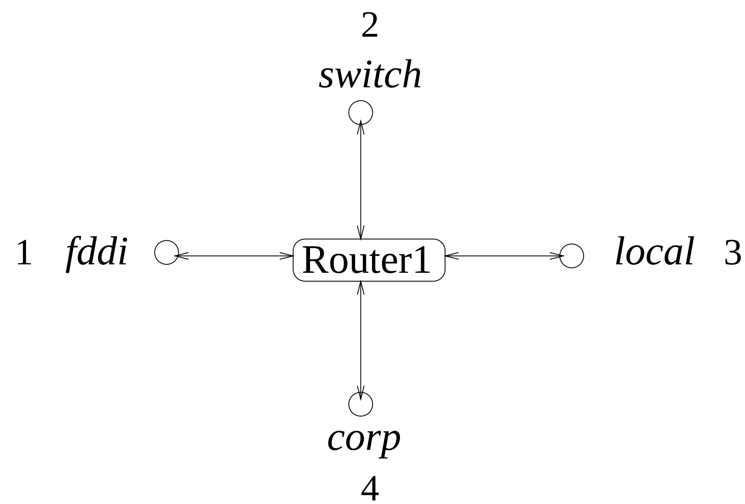
# An OD Matrix Estimation Example




- $n = 4$ edge nodes, 1 router;
- $J = n^2 = 16$ OD pairs in $X$;
- $I = 7$ independent links in $Y$.

| dst(fddi) | dst(switch) | dst(local) | dst(corp) | total |
|---|---|---|---|---|
| $4 \to 1$ | $4 \to 2$ | $4 \to 3$ | $4 \to 4$ | src(corp) |
| $3 \to 1$ | $3 \to 2$ | $3 \to 3$ | $3 \to 4$ | src(local) |
| $2 \to 1$ | $2 \to 2$ | $2 \to 3$ | $2 \to 4$ | src(switch) |
| $1 \to 1$ | $1 \to 2$ | $1 \to 3$ | $1 \to 4$ | src(fddi) |

# An OD Matrix Estimation Example (cont)




$$
\begin{pmatrix} \text{src(fddi)} \\ \text{src(switch)} \\ \text{src(local)} \\ \text{src(corp)} \\ \text{dst(fddi)} \\ \text{dst(switch)} \\ \text{dst(local)} \end{pmatrix}
=
\begin{pmatrix}
1&1&1&1& & & & & & & & & & & & \\
 & & & &1&1&1&1& & & & & & & & \\
 & & & & & & & &1&1&1&1& & & & \\
 & & & & & & & & & & & &1&1&1&1\\
1& & & &1& & & &1& & & &1& & & \\
 &1& & & &1& & & &1& & & &1& & \\
 & &1& & & &1& & & &1& & & &1&
\end{pmatrix}
\begin{pmatrix} 1 \rightarrow 1 \\ 1 \rightarrow 2 \\ 1 \rightarrow 3 \\ 1 \rightarrow 4 \\ 2 \rightarrow 1 \\ \cdots \\ 4 \rightarrow 3 \\ 4 \rightarrow 4 \end{pmatrix}
$$

# A Generalized Linear Network Tomography Model

The term *network tomography* was first used by Vardi (1996) to capture the similarities between the problem of OD matrix estimation through link counts and medical tomography.

A linear tomography model was devised by Vardi (1996) for OD traffic estimation. At a given time $t$, let $X$ be unknown quantity of interest (of dim $J$), $Y$ be known linear aggregations of $X$ (of dim $I$). We have

$$Y = AX,$$

where $A$ is the routing matrix with usually $J \gg I$, i.e., an ill-posed linear inverse problem.

The model is later generalized to other network tomography problems of similar properties (Coates, Nowak, Hero and Yu, 2002).

# An Incomplete Reference Review

- Statistical methods:
  - Vardi (1996): a Poisson traffic model;
  - Tebaldi and West (1994): a Bayesian perspective (MCMC implementation);
  - Cao et al. (2000): a Gaussian model w/ a power-law mean-var relationship;
  - Zhang et al. (2003): a regularized gravity model;
  - Liang and Yu (2003): a pseudo likelihood approach;
  - Vaton and Gravey (2003): an empirical Bayesian method.
- Direct measurement methods:
  - Feldmann et al.(2002): deriving traffic matrix through Netflow software.
- Hybrid methods:
  - Nucci et al. (2003): the use of routing changes to obtain more information about the underlying OD traffic;
  - Soule et al. (2005): a Kalman filter model with partial measurements.

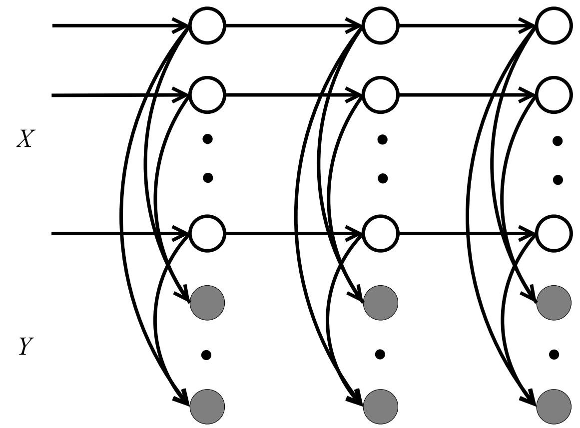
# A Partial Measurement Approach: Motivations

- Existing methods are not satisfactory in estimating OD traffic matrix:
  - Direct measurement approaches are too expensive;
  - Indirect statistical methods (only through links counts) are usually of bad performance: 20%-30% average relative errors;
- The recent discovery of the low (not seemly high) dimensionality of OD traffic (Lakhina et al., 2004) implies the possibility of using only a few measurements to characterize the OD traffic;
- Time series information is important for improving OD matrix estimation.

PamTram (PArtial Measurement for TRAffic Matrix estimation) combines direct and indirect approaches, providing a feasible way to explore the OD space dynamically.

# A Dynamic Network Traffic Model

The dynamic network model is a revised Gaussian traffic model in Cao et al. (2000) but time dependent structure:




Link Traffic: $Y^{(t+1)} = AX^{(t+1)}$

OD Traffic: $X^{(t+1)} = X^{(t)} + \epsilon^{(t)}$

$$\epsilon^{(t)} \sim N(0, \eta^{(t)} \text{diag}(X^{(t)}))$$

where

$\epsilon^{(t)}$: independent error terms with variance proportional to $X^{(t)}$

$\eta^{(t)}$: unknown positive quantities characterizing the network traffic dynamics; we assume $0 < \eta^{(t)} < \eta$.

- How to estimate $X^{(t)}$ with the $Y^{(t)}$ and additional observed OD traffic?
- How to choose informative OD pair(s) to measure?

# Error Metrics

- Mean square error (MSE): $\text{MSE}(\hat{X}, X) = ||\hat{X} - X||^2$.

- Scaled mean square error (sMSE)

$$\text{sMSE}(\hat{X}, X) = \frac{||\hat{X} - X||^2}{||X||_1} = \frac{\sum_i (\hat{X}_i - X_i)^2}{\sum_i X_i}.$$

The MSE metric does not scale to traffic volume changes.

- Relative error

$$\text{Rel-Error}(\hat{X}_i, X_i) = \frac{|\hat{X}_i - X_i|}{X_i}.$$

# Iterative Proportional Fitting

Given (a) a set of linear constraints $L$ (i.e., $AX = Y$ and extra measurements)
(b) a starting distribution $q$ (i.e., the previous estimate)
then the *I*-projection (Csiszar, 1974) of $q$ onto $L$ is defined as

$$\hat{p} = \arg\min_{p \in L} D(p||q).$$

- *I*-projection yields the maximum entropy solution when $q$ is uniform;
- Iterative Proportional Fitting (IPF) is a simple alternating minimization procedure to find the *I*-projection solution.

# IPF for Traffic Matrix Estimation

Theorem: For the network dynamic model, if $X^{(t-1)}$ is known, then the IPF estimate of $X^{(t)}$ is approximately the minimum mean square error (MSE) estimate:

$$E(X^{(t)}|Y^{(t)}) = X^{(t-1)} + A\Sigma(A\Sigma A')^{-1}(Y^{(t)} - AX^{(t-1)})$$

where $\Sigma = \eta^{(t)}\mathrm{diag}(X^{(t-1)})$.

- the solution is independent of parameter $\eta^{(t)}$;
- the IPF estimate is ensured to be non-negative, while the minimal MSE estimate is not.

# How to Select Informative OD Traffic Pairs?

We consider an extreme case of selecting only one OD pair to measure!

- The ideally criteria is to select the OD pair which will reduce next step MSE the most. We need to compute the conditional variance

$$\mathrm{Var}\left(X^{(t)} \middle| Z^{(1)}, \cdots, Z^{(t)}\right) \text{ where } Z^{(i)} \text{ is the observed data at time } i$$

  Too costly: it involves integrating over all past observations.

- Our solution is to adopt randomization rules to select informative OD pairs based on the current estimate. The idea is based on the estimate residual

$$R^{(t)} = X^{(t)} - E\left(X^{(t)} \middle| X^{(t-1)} = \hat{X}^{(t-1)}, Y^{(t)}\right).$$

  Goal: to select OD pairs with large absolute residuals to measure.

# Uniform Randomization Rule

**Theorem**: The uniform randomization rule is the minimax decision rule of the pick-largest-absolute-residual game with a 0-1 payoff function.

**Theorem**: Let $\mathrm{sMSE}^{(t)}$ be the scaled MSE error at step $t$. Assume only one OD pair is selected for measurement by uniform random sampling (no link measurements $Y$ are made), then the error metric of $\hat{X}^{(t)}$, is approximately bounded by

$$E(\mathrm{sMSE}^{(t)}) \leq \frac{I-1}{I}\mathrm{sMSE}^{(t-1)} + \eta,$$

- The uniform rule is a simple randomization scheme and easy to implement;
- It does not use other available information, hence not optimal;
- The minimax property implies its robustness to unexpected events.

# Maximum Entropy Randomization Rule

In practice, $\hat{X}^{(t-1)}$ should be close to the true $X^{(t-1)}$. If $X^{(t-1)} = \hat{X}^{(t-1)}$, then

$$R^{(t)} \sim N\left(0, \Sigma^{(t)} - \Sigma^{(t)} A' (A \Sigma^{(t)} A')^{-1} A \Sigma^{(t)}\right) \text{ where } \Sigma^{(t)} = \eta^{(t)} \mathrm{diag}(\hat{X}^{(t-1)}).$$

Define

$$Q(i) = P\left(|R_i^{(t)}| = \max_j |R_j^{(t)}|\right).$$

Maximum entropy (maxen) randomization scheme:

to pick the $i$th OD flow with probability $P_{maxen}(i) = Q(i)$.

- It is well known that $P_{maxen}$ is the maximum entropy solution with $-\log$ loss;
- The maxen randomization can be approximated by IPF algorithm!

# A Weighted Randomization Scheme

- The uniform rule is to explore the OD space
  - less efficient;
  - more robust to sudden network traffic changes.
- The maxen rule is to exploit the previous estimate
  - more efficient;
  - more sensible to traffic changes and model assumption.

A weighted maxen (wMaxen) randomization rule is defined as:

$$P_{wMaxen} = \alpha P_{uniform} + (1 - \alpha) P_{maxen}.$$

where $\alpha \in (0, 1)$ is a preset parameter. We usually set it as a relative small number, such as $0.2$, to favor the maxen method.

# The PamTram Randomization Algorithm

Initialization: Set $\hat{X}_0 = \mathbf{1}$

**for** time interval $t$ **do**

1. Measure OD pairs selected at step $t-1$;
2. Estimate $X^{(t)}$ by applying IPF algorithm to $\hat{X}^{(t-1)}$ (with constraints);
3. Determine OD pairs to measure at $t+1$ by using randomization rules.

**end for**

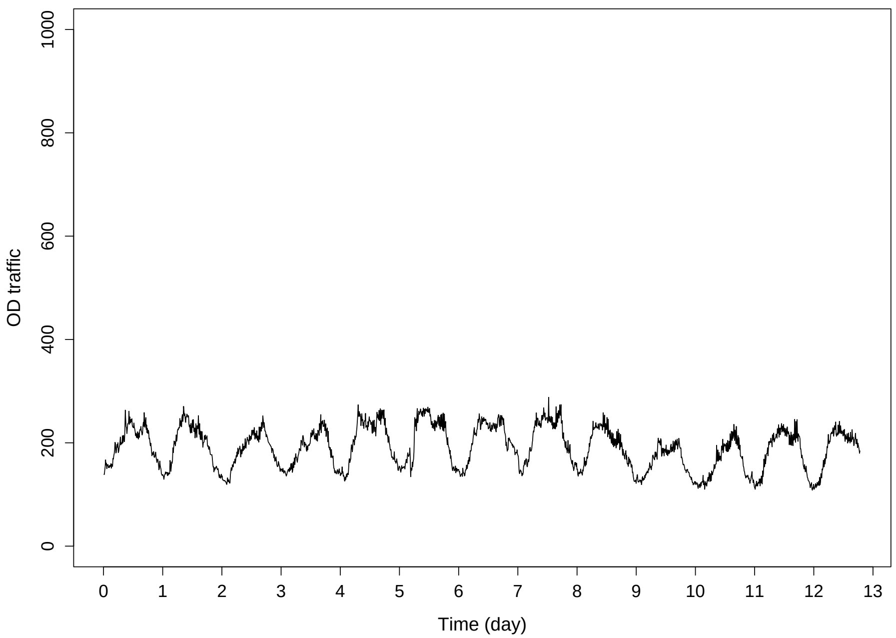
The initialization $\hat{X}_0 = \mathbf{1}$ implies we start with a maximum entropy estimation.

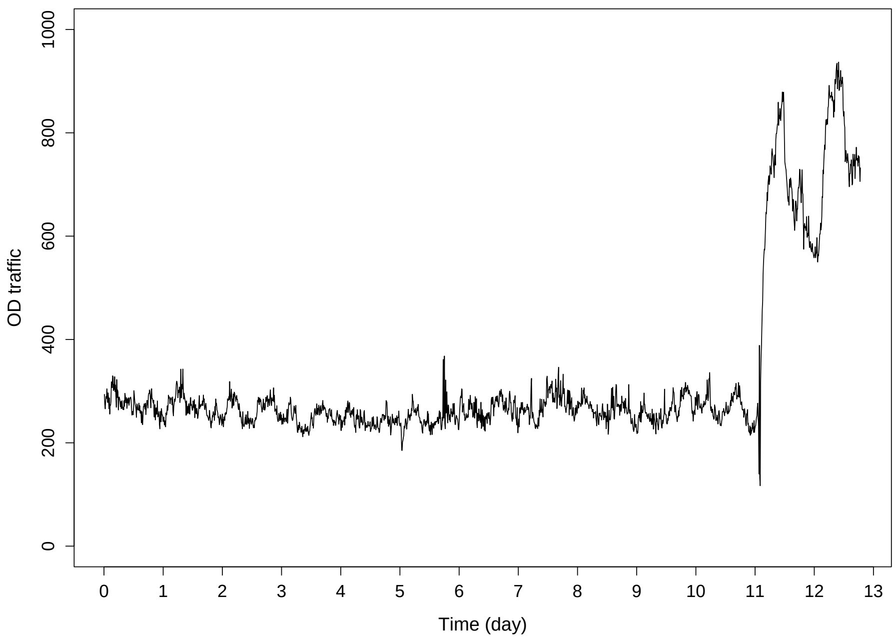
# Computational Complexities

Each round of the proposed PamTram algorithm requires at most two IPF procedures for maxen and wMaxen, and only one IPF for the uniform randomization rule.

- IPF algorithm is easy to implement;
- It has a fast quadratic convergence rate (Liang and Yu, 2004);
- The algorithm is further speeded up because the starting point is usually within a small neighborhood of the final solution.

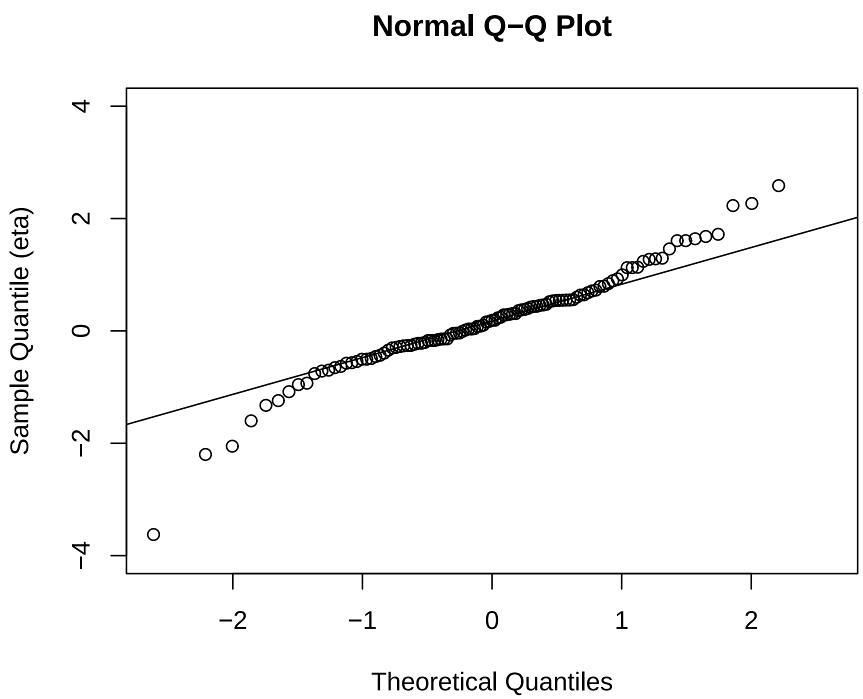
# Sprint European PoP Network Data

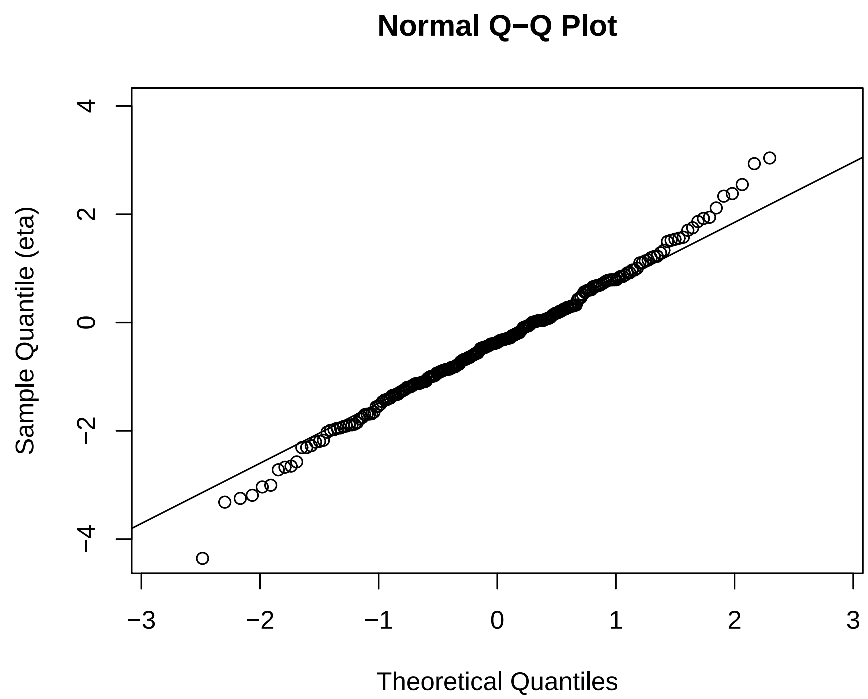



- The validation data, i.e., the ground truth, is available;
- Periodicity of network traffic is evident;
- The mean OD traffic has slow variability;
- The traffic is smooth most of time.

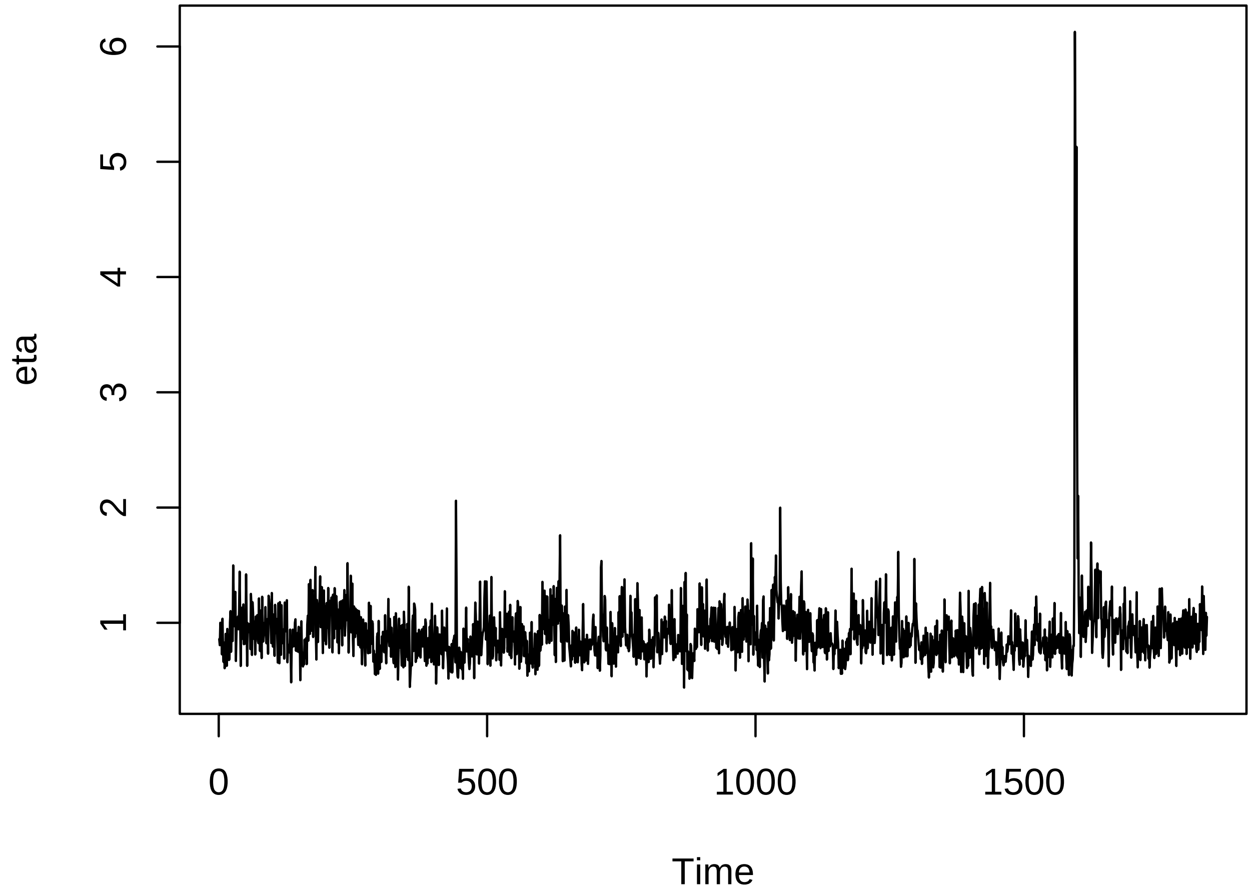
# Model Checking — Normality




$$U_{t,i} = \frac{X_i^{(t)} - X_i^{(t-1)}}{\sqrt{X_i^{(t-1)}}} \sim N(0, \eta^{(t)}).$$

# Model Checking — $\eta^{(t)}$ Plot




Method of moment (absolute moment) estimate of $\eta^{(t)}$.

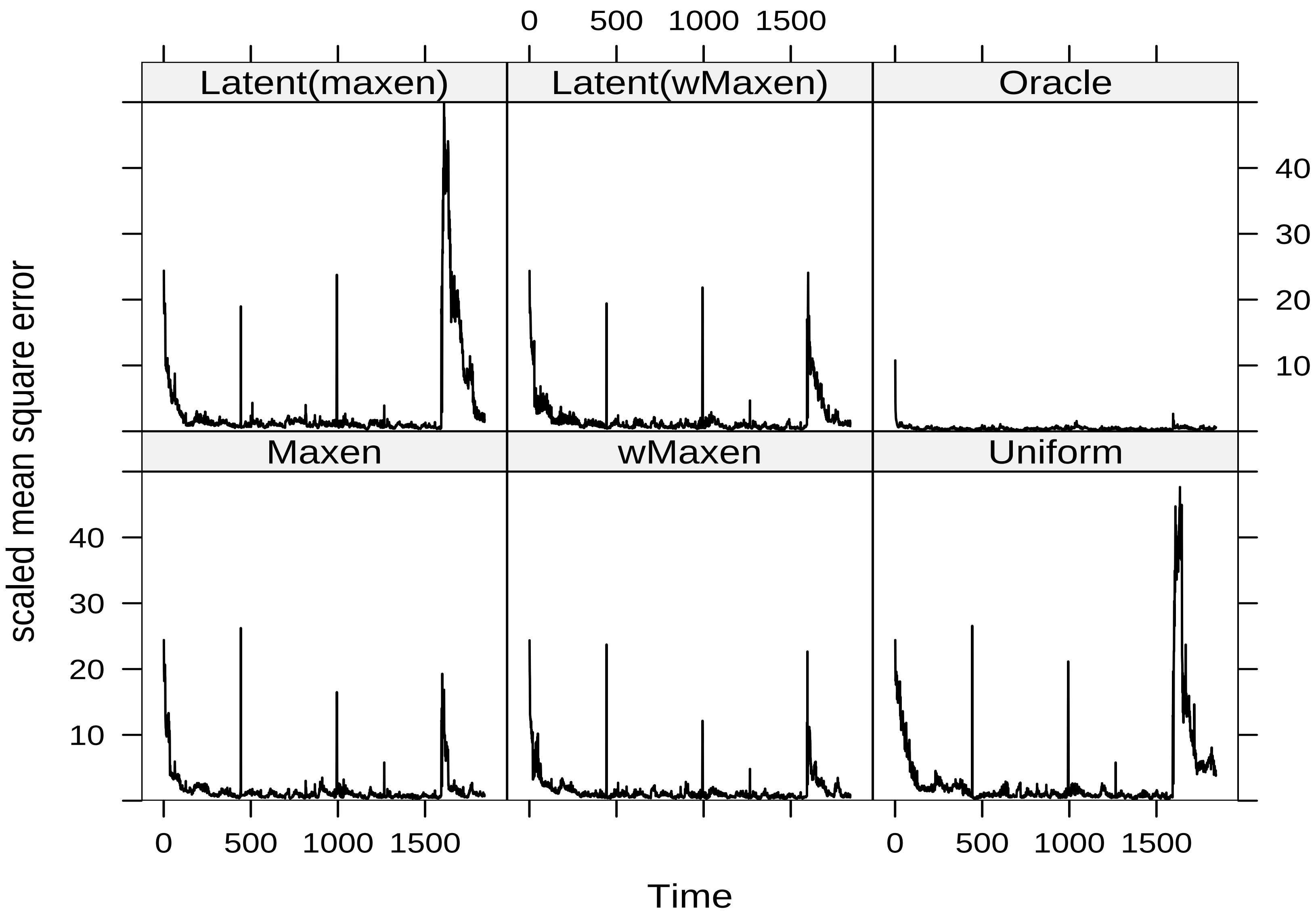
# More Selection Schemes

In order to have an in-depth comparison, we also implement two other types of OD selection schemes:

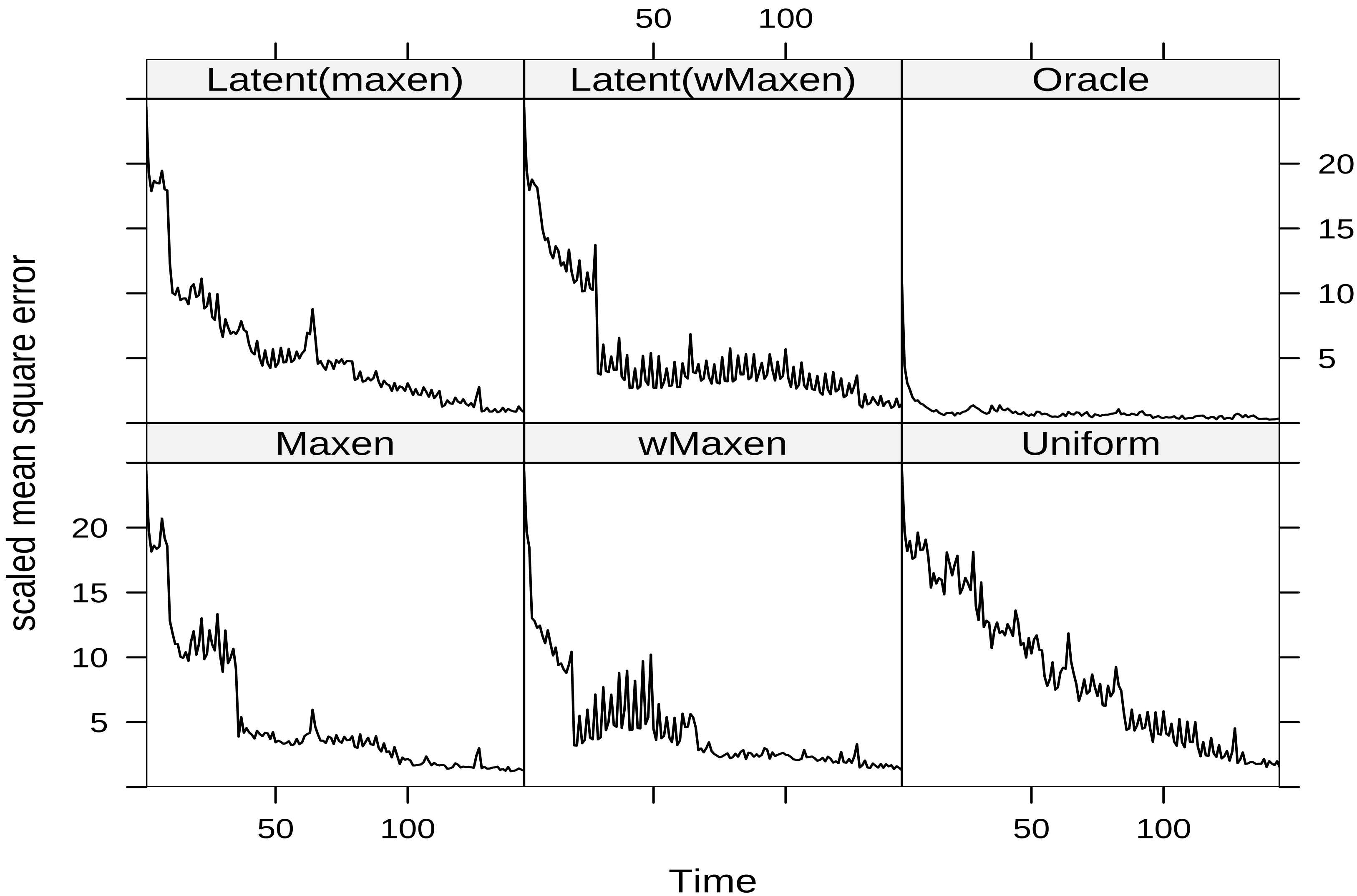
- Oracle: pick the OD pair to measure such that it results in the smallest next step scaled MSE;
- Latent: a scheduling meta scheme. The OD pairs selected will be really measured 24 hours later. The lead time can be used for the network to disseminate and schedule the monitoring activities.

In total, six randomization schemes are implemented for this dataset: oracle, uniform, maxen, wMaxen, latent(maxen), latent(wMaxen).

# The Full Scaled-MSE Plot

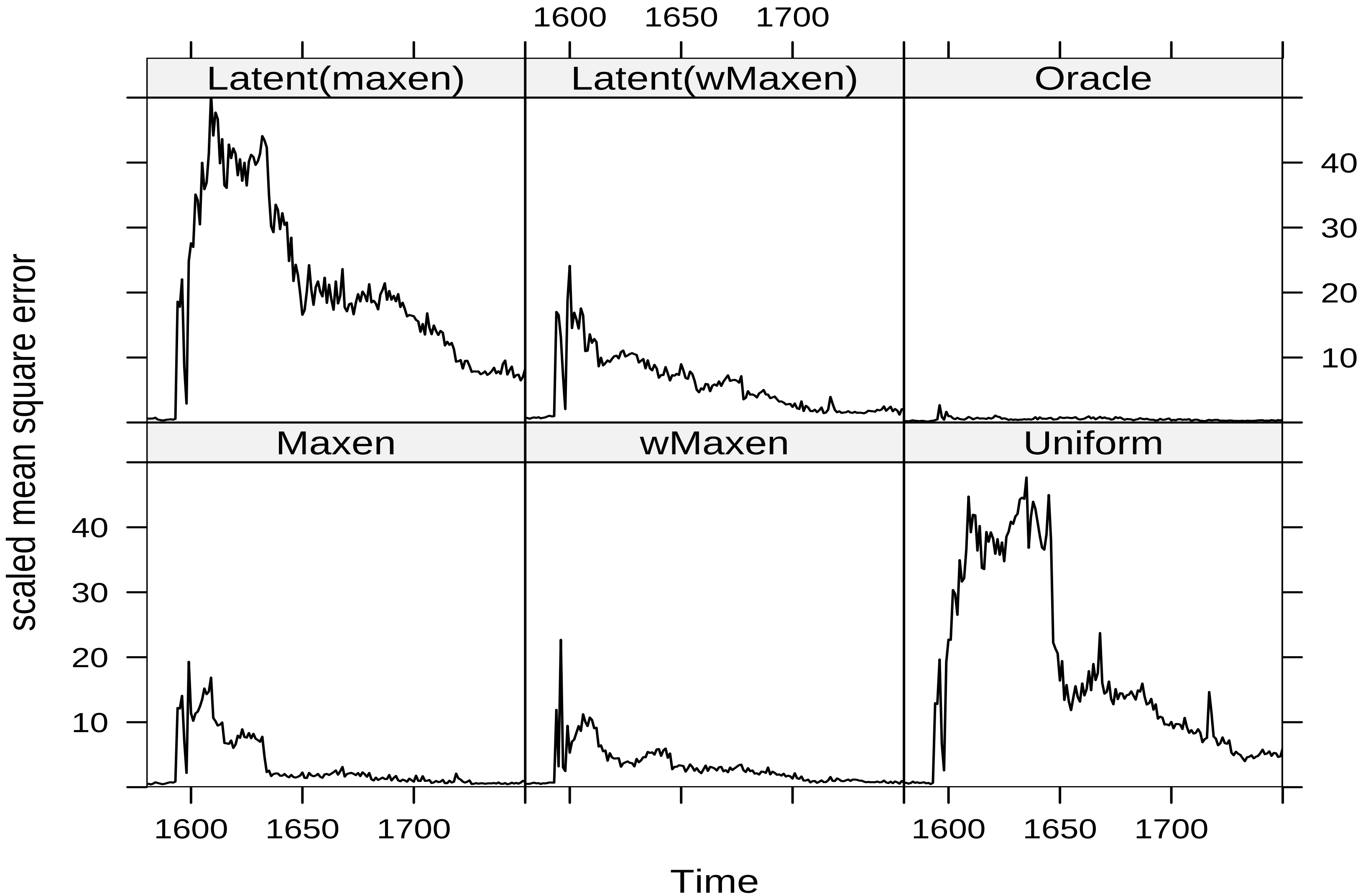


# Zoomed-in Scaled-MSE Plot (I)




Zoomed-in scaled-MSE plot – the starting region.

# Zoomed-in Scaled-MSE Plot (II)




Zoomed-in scaled-MSE plot: the region with a large sudden traffic change.

# Relative Error and Running Time

| | Avg(relError) | Avg(sMSE) | Time (sec/interval) |
|---|---|---|---|
| Oracle | 4.4% | 0.42 | n/a |
| Maxen | 7.5% | 1.57 | 0.31 |
| wMaxen | 7.5% | 1.45 | 0.28 |
| Latent(maxen) | 9.2% | 3.35 | 0.27 |
| Latent(wMaxen) | 7.9% | 1.79 | 0.25 |
| Uniform | 9.4% | 3.50 | 0.15 |
| Tomogravity | 27% | n/a | 15.6 |

The dataset is processed on a 3.2GHZ computer using the *R* package.
The *tomogravity* method is implemented using matlab.

# Conclusions

- The hidden Markov structure introduced by the dynamic network traffic model is of importance in the PamTram approach. It is possible to incorporate further network dynamics into such a model to make it even more realistic;

- The idea of the adaptive partial measurement approach is powerful and has the potential to be generalized to many other problems.